NEW SENSOR GEOMETRIES FOR IMAGE PROCESSING:
COMPUTER VISION IN THE POLAR EXPONENTIAL GRID

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ABSTRACT

This paper provides a capsular introduction to the theoretical framework and experimental applications of the *Polar Exponential Grid* (PEG) transformation, in the context of image analysis. The PEG transformation is an *isomorphic* (1) representation of the image intensity array that simplifies, and potentially offers new insights about, a variety of tasks in computational vision. We describe the PEG transform representation; we briefly survey its functional precursors in optical computing and image processing. We then give an example of PEG-based image analysis for rotation-and-scale variant template matching and, present the PEG transform as a motif for a class of problems in stochastic estimation of object boundaries.

INTRODUCTION TO THE POLAR EXPONENTIAL GRID (PEG)

Retrospective

This paper addresses the related questions of novel focal-plane geometries and new sensory data representations to expedite tasks in low-level image analysis. The Cartesian grid structure of most image sensors is the happenstance of technological convention in integrated circuit engineering and machine architecture. We seek to understand if potential alternatives to the accepted standard offer improved system throughput for machine vision. By serendipity, might such alternative data representations offer clues to new image processing computer architectures deviant from the norm and better suited to image understanding vis-a-vis 2-D signal processing *per se*?

Our interest in this problem was motivated by our earlier studies (2, 3) in theory, models, and parallel algorithm design for high speed image segmentation. As we shall later discuss, our work therein dealt with extraction of simple-object boundaries from noise-degraded imagery. In this, and a far more general class of computational vision problems, one must deal with problems of variable object scale, orientation, aspect, and illumination/radiation, among others. We in particular sought an alternative to Cartesian array image representation that would provide some invariance against the first two problems commonly encountered in overhead views typical of production line robotics and terrestial surveillance problems. We considered not only potentially new ways of acquiring sensory data, but the implications the corollary image representation would have for the structure of our segmentation algorithms. A prerequisite for any new representation was that it be *isomorphic*(1) to the sensory data: that is, in the sense of our segmentation algorithms, an image representation other than the conventional intensity array must be attribute-preserving. This meant for our needs, and to serve a far broader class of 2-D and 3-D segmentation problems, a new representation should be iconic versus symbolic, and preserve both global image topology and local geometry (viz., angles and sense of rotation). Further requirements particular to our view of 2-D segmentation were that the new image representation should: 1) be a natural motif for the pseudo- 1-dimensional functional descriptions appropriate to some classes of closed boundaries (5); and 2) provide a format for a scale-invariant (*resolution-preserving*) boundary description. "1)" is very desirable in the formulation of a class of effective stochastic boundary estimators (4, also ref. [8] in 2). "2)" overcomes for some applications theoretical problems in boundary modeling and, ameliorates model implementation problems of spatial quantization error.

PEG Geometry and Precursors in Image Processing and Display

The Polar Exponential Grid transformation is described by Fig. 1. The relationship between the grid and its data representation is a conformal mapping (6); this point is most important because it obtains the aformentioned requirement of invariant global topology and local geometry in the mapping of sensory data to its representation. Weiman and Chaikin in their study (7) of a more general class of iconic image representations that include the PEG, have begun to explore some of the implications of conformal computational geometry for its *dual* applications to image analysis and computer graphics. In particular, their work along with ours recognizes the potentially attractive properties of special purpose machine architectures deriving from conformal data representations.

It should be apparent at this point that PEG transformation reduces scale and rotation variants to translations in the isomorphic representation. Further, for pixel edge-element-based boundary descriptions, the PEG representation is resolution-preserving; also, vis-a-vis the sensory data, the image representation is comatic. For 2-D linear filtering operations on its representation, the PEG mapping is one of a class of invertible transformations that meet the definition of the *Coordinate Transform Processor* (9) depicted in Fig. 2. Robbins and Huang (10), and Sawchuk (11,12) exploited the PEG transformation in this context to effect space-variant filtering procedures for digital restoration from comatic aberration, and motion blur. A continuous paradigm to the PEG representation has received recent consideration in the context of coherent optical computing by Casasent and Psaltis (13). Their work has conceptual ties to that of ours and that of Weiman and Chaikin, in that Casasent and his colleagues seek among others a correlator architecture that is robust against variations in input image scale and rotation. Their proposed system achieves this by concatenated operations on image Fourier amplitude spectra. As alluded in (7), this strategy does not maintain isomorphism due to loss of phase information.

EXPERIMENTAL RESULTS

In this section we present results for selected experiments in PEG-based image analysis; more extensive reports follow in (14, 17). We first outline an autocorrelation study which is illustrative of attentive *(foveal)* PEG vision. We then discuss in more detail the application of PEG representation to stochastic boundary estimation.

PEG Transforms and Template Registration

Fig. 3 a) is an example of PEG transformation and data reconstruction (16) applied to the General Motors Corp. "bin-of-parts" data base. It was earlier noted that the PEG representation is a natural framework for fast linear scale-and-rotation cross-correlation. Fig. 3b) depicts results for a 2-D autocorrelation study on an object (inset) extracted from the data base. The autocorrelation function can be seen to have a well-defined principal maximum for both shift coordinates. Meaningful cross-correlation tests presume the ability to translationally center, viz., *foveate* on, the object. This is a subtle issue with implications to both 2- and 3-dimensional segmentation; we explore this question in refs. (15) and (17) (see also ref. 8) and offer related remarks in the next subsection.

Simple-Object Segmentation

In references (2, 3) we introduced a new class of parallel-window estimator for simple-object boundary extraction from noisy data. A representative algorithm, the *Parallel Hierarchical Ripple Filter*, was demonstrated for application to a Cartesian array representation of the sensory data. Here, we demonstrate application of this algorithm to the PEG representation.

Our approach to boundary extraction derives from Cooper (2, refs. [3, 4]), and earlier, Nahi (ibid., ref. [8]). Stochastic models, e.g. Fig. 4, are defined for the data generation processes; then, boundary identification is realized as a search of the boundary edge-element parameter space to obtain a maximum *a posteriori* likelihood estimate for the data-and-boundary model joint probability functional. An exhaustive boundary search is computationally prohibitive for even small (32 x 32) data bases. Thus, a focus in the practical implementation of MLE boundary finding has been identification of viable suboptimal non-exhaustive search strategies and, their theoretical (4) and experimental (2 - 4) extension to concurrent computation.

The PHRF boundary estimator is a hierarchical resolution concurrent search algorithm. It partitions an (N' x N') data base to windows (N x N) whose edge dimensions are on the order of the boundary model correlation length. The algorithm conducts statistically independent, progressively resolved searches of each partition, and at higher (or the highest) resolutions infers statistical correlation of adjacent boundary estimates by concurrently smoothing across window partitions after independent search. The PHRF converges in log-time vis-a-vis search at full resolution.

Fig. 5 represents a scenario and experimental results for PHRF realization in the PEG representation. Arguments for such a PEG-based versus Cartesian PHRF realization are as follows: First, PEG boundary representation is resolution-preserving in scale vs. equi-resolved; thus, it provides a motif for a boundary search whose computational complexity is logarithmic vs. linear in domain dimensionality. Second, consider the result of Fig. 5 d): an interesting class of object images, e. g. computed-tomography organ scans, biological cells, clouds, FLIR targets, etc. are amenable to modeling in the PEG representation as cyclostationary 1-D processes. Taken in context with the first point, this suggests possibilities for a new class of highly structured (real-time implementable) robust boundary estimators. Finally, isomorphism of the PEG representation guarantees that segmentation procedures which derive from local geometric constraints will remain translationally invariant in the PEG representation. Contrasted with the previous point which addressed attentive vision, this point argues for the computational equivalence of segmentation in the central and peripheral fields of the PEG representation. Control issues of hierarchical machine vision that bear on this PEG attentive (foveal) - global (peripheral) paradigm to biological vision are discussed in refs. (15) and (17). Such questions are apropos to the aforementioned problem of translational template registration. In essence, we believe that inroads to efficiently solving segmentation problems will come not only from appreciation of the control structures involved, but also from insights provided by the computational geometry of isomorphic representations (1, 8, 15).

SUMMARY

We have presented experimental examples of image analysis in the context of a new sensory data representation, the Polar Exponential Grid. Specifically, we have introduced a new class of boundary finding algorithm that has attributes of speed and robustness for large area search. We have made recommendations for further study of this problem. In generality, we argue for the philosophy that substantial progress in machine vision will arise not only through generalization of systems , but also through fundamentally different ways of representing sensory information and developing computational paradigms to exploit these representations. This in turn suggests consideration of new devices and machine architectures for vision. To offer advantages over the prevailing technological metaphors for biological vision, these isomorphic equivalents must make image and scene representation more parsimonius of both structure and computation. We believe that PEG representation is the kernel of one such new approach to machine vision.

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(18) Ref. 12, pg 140.

Figure 1 the Polar Exponential Grid and its representation; a lower bound is set on radius because of the singularity at r = 0. After ref.(7).

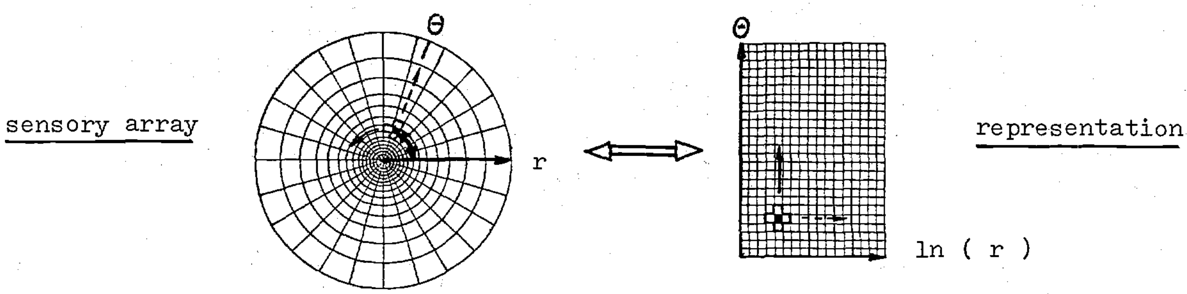

Figure 2 Coordinate Transform Processor(after ref.(9)). u,v correspond to angle and ln-radius of the Polar Exponential Grid. Linear shift-invariant filtering is performed on the PEG representation. x,y are the image display reconstruction. Applications are discussed in refs.(10-12); see also refs.(14-16) for experimental examples.

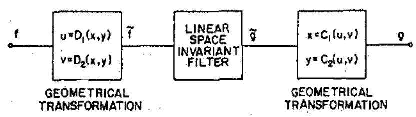

Figure 3 an experimental demonstration of PEG representation and image reconstruction(ref.(16)). Note the comatic loss of resolution in peripheral field of reconstruction. The rightmost figure depicts an autocorrelation study on the inset data base: horizontal axis is rotation; vertical axis is scale. Intensity is directly proportional to the local value of the autocorrelation function; the center of the intensity map corresponds to perfect registration. The superposed plot is autocorrelation vs. angle at 1:1 scale,i.e.,intensity along the horizontal axis. note: plot and map are not co-centered on horizontal axis(see scale marks for plot at bottom of picture).

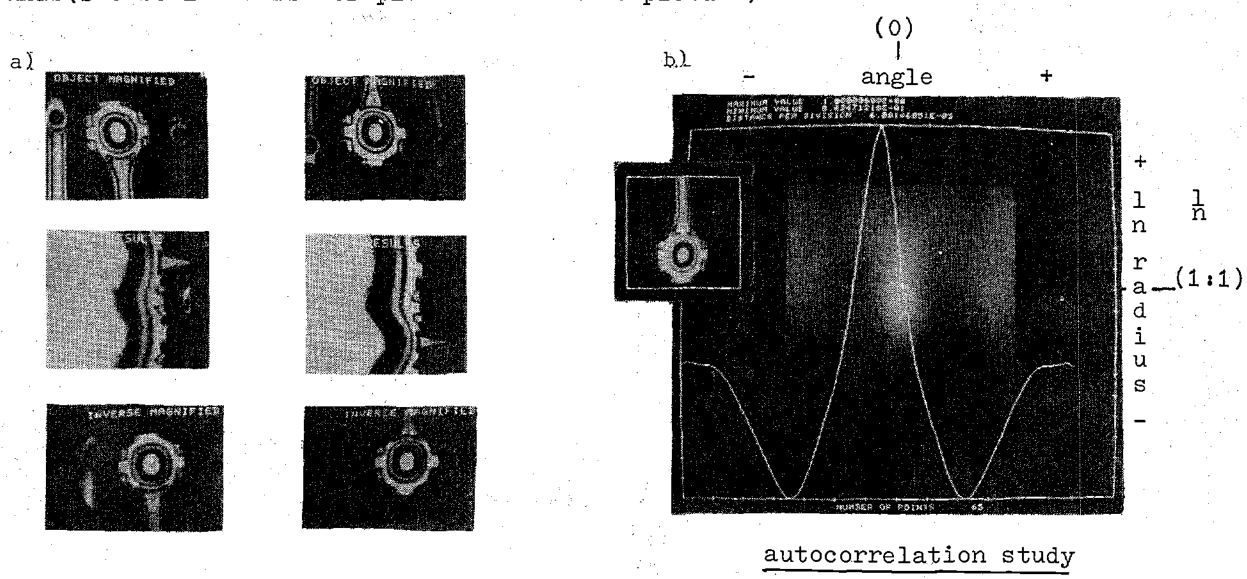

Figure 4 digitized data generation models for PHRF stochastic boundary estimation. Parameters r_{in}, r_{out} are the interior, exterior region expectation values for the picture function b_{jk}. n_{jk} is an additive uncorrelated Gaussian noise field. Signal-to-noise is defined as the ratio of $(r_{in}-r_{out})$ to the standard deviation of n_{jk}. The boundary is modeled as a K-th order Markov process, for which a state is K sequentially connected edge elements. The figure illustrates possible state transitions from $t_{i-1} \rightarrow t_i$ for a 7-th order model. Local boundary curvature is measured by angle theta, whose distribution one designs to reflect expectation of the local boundary structure. Here, boundary cost is designed to monotonically increase with theta, viz., the estimator will reflect a global bias for boundaries of low curvature. After refs. (2,3).

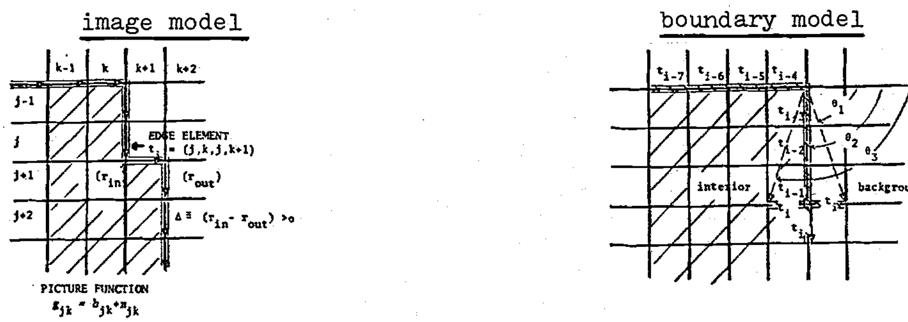

Figure 5 experimental results for PEG-based Parallel Hierarchical Ripple Filter operation:

a) 2:1 S/N circle-with-projection data base
b) partitioned PEG representation
c) parallel search of partitions in progress
d) contour estimate
e) reconstructed PEG image and estimated boundary
f,g) applications to boundary finding in forward -looking- infrared(FLIR) tank image, and to cloud image, per e)